Test 2 Practice Key, Math 1410

Spring 2013, Dr. Graham-Squire

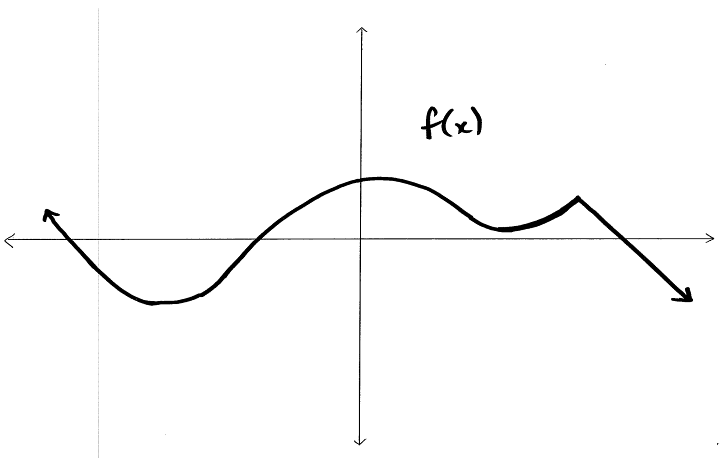
1. A particle moves along a horizontal line so that after t seconds its position is given by

$$s(t) = \frac{5}{3}t^3 - 10t^2 + 15t$$

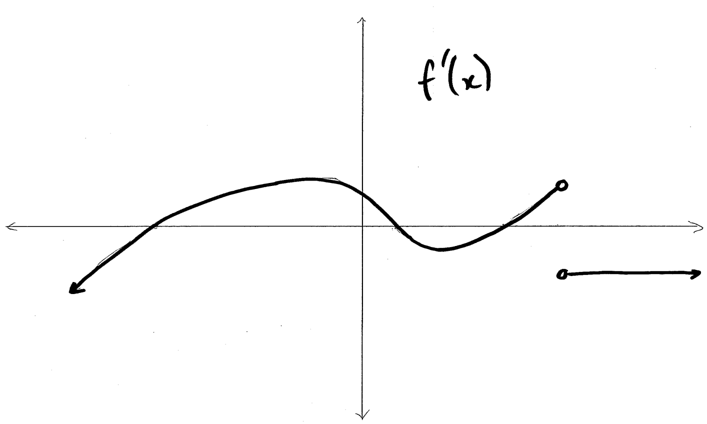
When is the particle moving left? Note: positive direction is to the right, so increasing = moving right. (Also, you should use the derivative rules to solve this question, you do not have to use the definition of the derivative).

Ans: The particle is moving left on the interval $(1, 3)$

2. Sketch a graph of $f'(x)$ if $f(x)$ is the graph given below:



Ans:



3. Use the limit definition of the derivative to calculate $\frac{d}{dx}(\sqrt{x-7})$.

Ans: $\frac{d}{dx}(\sqrt{x-7}) = \lim_{h \to 0} \frac{1}{h}(\sqrt{x+h-7} - \sqrt{x-7}) = \lim_{h \to 0} \frac{1}{h} \frac{x+h-7-(x-7)}{\sqrt{x+h-7}+\sqrt{x-7}} =$ $\lim_{h \to 0} \frac{1}{\sqrt{x+h-7}+\sqrt{x-7}} = \frac{1}{2\sqrt{x-7}}$

4. Find y' if $y = \dfrac{e^{-x}\cos x}{\ln x}$

   Ans: $\dfrac{(\ln x)(-e^{-x}\cos x - \sin x e^{-x}) - (1/x)(e^{-x}\cos x)}{(\ln x)^2}$

5. Find the x-coordinate(s) when the given function has a horizontal tangent line

$$T(x) = x^2 e^{1-3x}$$

   Ans: $x = 0$ and $x = 2/3$

6. Find $\dfrac{dy}{dx}$ if $y = \sqrt[3]{x + \sqrt{2\sec x}}$

   Ans:

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{2}(2\sec x)^{-1/2} 2\sec x \tan x)$$

   or

$$y' = \frac{1}{3}[x + (2\sec x)^{1/2}]^{-2/3}(1 + \frac{1}{\sqrt{2}}(\sec x)^{1/2}\tan x)$$

7. Find y' if $\ln(xy) = e^{2x}$

   Ans: $y' = 2ye^{2x} - \dfrac{y}{x}$

8. Calculate $\dfrac{d}{dx}\tan(\arctan x)$ two different ways- First take the derivative and then simplify your answer. Next, simplify the expression first and then take the derivative. (Hint: $\cos(\arctan x) = \dfrac{1}{\sqrt{1+x^2}}$....do you know how to prove it?)

   Ans: You should get 1 both ways you do it.